**Linear Functions**

A linear function is a function whose graph is a line. Linear functions can be written in the **slope-intercept form**:

where is the initial or starting value of the function (when input, ), and is the constant rate of change, or slope of the function. This is interpreted as the change in output per unit increase in input.

Example 1: The pressure, , in pounds per square inch (PSI) on a diver depends on the depth below the water surface, , in feet. This relationship may be modeled by the equation:

Interpret the values of 0.434 and 14.696 in context.

**Slope**

Given two values for input and two corresponding values for output, the slope of a linear function can be expressed and calculated using the following:

Example 2: If is a linear function with and Find the slope of and use it to determine if is increasing, decreasing, or constant.

Linear functions can also be written in the **point-slope form**:

where is the slope of the line and and are the coordinates of a specific point through which the line passes. This form is derived from the slope formula above. Can you determine how?

Example 3: Suppose we are told that a linear function has a constant rate of change of 2, and passes through the point . Write the equation of the line in point-slope form, rewrite it in slope-intercept form, and finally sketch the graph.

Example 4: Suppose we know a linear function passes through the points and Determine the constant rate of change, write the equation of the function in point-slope form, rewrite it in slope-intercept form, and finally sketch the graph.

Example 5: Write an equation for the linear function given the graph of below.

A graph of x and y axis

Description automatically generated

Example 6: Match each function with its graph.

A graph of a line graph

Description automatically generated

**Horizontal and Vertical Lines**

A vertical line indicates a constant input, or –value. Since one input has many outputs associated with it, a vertical line is NOT a function.

A horizontal line indicates a constant output, or –value. Since all inputs have the same output, the change in output is always zero .

or

NOTE: This is equivalent to

A graph of a function

Description automatically generated A graphing of a function

Description automatically generated

**Parallel and Perpendicular Lines**

Parallel lines in the two-dimensional plane will never intersect. Thus, they have the same slope and different –intercepts.

If and , then and are parallel if and .

Perpendicular lines in the two-dimensional plane do intersect, and form a right angle at the point of intersection. The slopes of perpendicular lines are negative reciprocals of each other, which is the same as saying the product of the slopes equals 1.

If and , then and are perpendicular if , or .

Example 7: Find the line parallel to the graph of and passes through the point .

Example 8: Find the line perpendicular to the graph of and passes through the point

**Average Rate of Change**

A rate of change describes how an output quantity changes relative to the change in the input quantity. The units on a rate of change are “output units per input units.”

The average rate of change between two input values is the total change of the function values (output values) divided by the change in the input values.

Example 9: The electrostatic force measured in newtons, between two charged particles can be related to the distance between the particles in centimeters, by the formula

Find the average rate of change of force if the distance between the particles is increased from 2 cm to 6 cm.

**Modeling with Linear Functions**

When modeling scenarios with linear functions and solving problems involving quantities with a constant rate of change, we typically follow the same problem strategies that we would use for any type of function. Let’s briefly review them:

1. Identify changing quantities, and then define descriptive variables to represent those quantities. When appropriate, sketch a picture or define a coordinate system.
2. Carefully read the problem to identify important information. Look for information that provides values for the variables or values for parts of the functional model, such as slope and initial value.
3. Carefully read the problem to determine what we are trying to find, identify, solve, or interpret.
4. Identify a solution pathway from the provided information to what we are trying to find. Often this will involve

checking and tracking units, building a table, or even finding a formula for the function being used to model the

problem.

1. When needed, write a formula for the function.
2. Solve or evaluate the function using the formula.
3. Reflect on whether your answer is reasonable for the given situation and whether it makes sense mathematically.
4. Clearly convey your result using appropriate units, and answer in full sentences when necessary.

Example 10: There is a straight road leading from the town of Westborough to Agritown 30 miles east and 10 miles north. Partway down this road, it junctions with a second road, perpendicular to the first, leading to the town of Eastborough.

Draw a visual to help represent the information above.

If the town of Eastborough is located 20 miles directly east of the town of Westborough, how far is the road junction from Westborough?